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A STUDY OF VARIOUS ANALYTICAL
TECHNIQUES FOR THE INFLIGHT
COMPENSATION OF GYRO DRIFT RATES

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FOREWORD

This report presents the results and conclusions obtained from a study of various proposed analytical techniques for the inflight compensation of gyro drift rates. It was prepared in compliance with Contract NAS3-3232.

SUMMARY

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Four proposed methods to eliminate inflight gyro torquing for gyro drift compensation were examined for equation accuracy and airborne-computer programming requirements. The results in terms of inflight-storage and compute-cycle penalties show that the drifted rotating-platform coordinate-system and quasi-linear matrix-transformation methods were best. The accuracy of the latter is limited by initial assumptions; therefore, the drifted rotating-platform coordinate-system method, being an "exact" method, was recommended.

author

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INTRODUCTION

The purpose of this study is to perform a detailed analysis of several proposed schemes[1] for eliminating inflight gyro torquing as a means of gyro drift compensation. The discussions are limited solely to the software aspects of the problem, i.e., guidance equation accuracy and the airborne-computer programming requirements.

The various schemes analyzed are designated as:

Method 1. Matrix transformation.

Method 2. Drifted rotating-platform coordinate system.

Method 3. Rotating t, n, r coordinate system.

Method 4. Guidance parameter biasing.

Note that Methods 1, 2, and 3 are explicit whereas Method 4 is not. Methods 1, 2, and 4 were programmed in "engineering" closed-loop guidance simulations on the IBM 7094 computer. Only the airborne-computer programming aspects (storage requirements and effect on length of compute cycle) of Method 3 were appraised.

The following sections present detailed discussions on the theory and application of each of the four methods for the analytical compensation of gyro drift.

[1] GD/C Memo CGA-319, Status of Inflight Torquing Techniques Study, 12 January 1965

SECTION 1

MATRIX TRANSFORMATION

1.1 DISCUSSION. The fundamental technique employed here is simply to transform the thrust acceleration vector from the rotating drifted-platform axes (where it is sensed) to inertial axes by means of a coordinate transformation matrix M. Thus, the thrust acceleration in inertial coordinates is given by

$$a_T = M a'_T \tag{1-1}$$

where, a_T, thrust acceleration in inertial coordinates (u, v, w axes), is

$$a_T = \begin{bmatrix} a_{Tu} \\ a_{Tv} \\ a_{Tw} \end{bmatrix}$$

a'_T, thrust acceleration in rotating drifted platform coordinates (u′, v′, w′ axes), is

$$a'_T = \begin{bmatrix} a_{Tu'} \\ a_{Tv'} \\ a_{Tw'} \end{bmatrix}$$

and M, coordinate transformation matrix, is

$$M = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix}$$

Integrating Equation 1-1, the thrust velocity is obtained.

$$v_T = \int_o^t M(t)\, a'_T(t)\, dt \tag{1-2}$$

The change in thrust velocity between time instants t_{i-1} and t_i is

$$\Delta v_{Ti} = \int_{t_{i-1}}^{t_i} M(t)\, a'_T(t)\, dt \qquad (1\text{-}3)$$

Integrating Equation 1-3 by parts,

$$\Delta v_{Ti} = M v'_T \Big]_{t_{i-1}}^{t_i} - \int_{t_{i-1}}^{t_i} \frac{dM}{dt} v'_T\, dt \qquad (1\text{-}4)$$

where

$$v'_T = \int_o^t a'_T\, dt$$

Evaluating Equation 1-4 and using the relations

$$M(t_{i-1}) = M(t_i) - \Delta M_i$$

$$\Delta v'_{Ti} = v'_T(t_i) - v'_T(t_{i-1})$$

then,

$$\Delta v_{Ti} = M(t_i)\, \Delta v'_{Ti} + \Delta M_i\, v'_T(t_{i-1}) - \int_{t_{i-1}}^{t_i} \frac{dM}{dt} v'_T\, dt \qquad (1\text{-}5)$$

Using the mean value theorem of the integral calculus, the last term on the right may be expressed as

$$\int_{t_{i-1}}^{t_i} \frac{dM}{dt} v'_T\, dt = \frac{dM}{dt}(t_\xi)\, v'_T(t_\xi)\, \Delta t_i \qquad (1\text{-}6)$$

where $t_{i-1} \leq t_\xi < t_i$ and $\Delta t_i = t_i - t_{i-1}$. Note that there are actually three time instants, t_ξ , corresponding to each of the three independent components of the vector implied by Equation 1-6. However, this is of no significant consequence as will be seen below. Now since

$$\frac{dM}{dt}(t_\xi)\, \Delta t_i \simeq \Delta M_i$$

Equation 1-5 may be written as

$$\Delta v_{Ti} = M(t_i)\,\Delta v'_{Ti} + \boxed{-\Delta M_i\left[v'_T(t_\xi) - v'_T(t_{i-1})\right]} \qquad (1\text{-}7)$$

The term enclosed by the dashed box will be negligible for reasonably small values of gyro drift rate and compute cycle length. For example, assuming a constant drift rate of 5 deg/hr and a 1.5 sec compute cycle, the maximum possible contribution (accumulated) for this term is less than 1 ft/sec! Thus, the approximation

$$\Delta v_{Ti} = M(t_i)\,\Delta v'_{Ti} \qquad (1\text{-}8)$$

is sufficiently accurate.

The M matrix is obviously related to the gyro drift rates. This relationship can be found as follows. Differentiating both sides of Equation 1-1 with respect to time,

$$\dot{a}_T = \dot{M} a'_T + M \dot{a}'_T \qquad (1\text{-}9)$$

Alternately, the vector $\dot{a}_T$ may be calculated as

$$\dot{a}_T = M(\dot{a}'_T + \Omega a'_T) \qquad (1\text{-}10)$$

where

$$\Omega \triangleq \begin{bmatrix} 0 & -\omega_{dw'} & \omega_{dv'} \\ \omega_{dw'} & 0 & -\omega_{du'} \\ -\omega_{dv'} & \omega_{du'} & 0 \end{bmatrix} \qquad (1\text{-}11)$$

and $\omega_{du'}$, $\omega_{dv'}$, and $\omega_{dw'}$ are the instantaneous drift rates about the u′, v′, and w′ axes respectively.

Eliminating $\dot{a}_T$ from Equations 1-9 and 1-10,

$$(\dot{M} - M\Omega)\, a'_T = 0 \qquad (1\text{-}12)$$

Now, since this result is valid for any arbitrary a'_T,

$$\dot{M} = M\Omega \qquad (1\text{-}13)$$

Furthermore, since M is relatively constant during a single compute cycle (at least in the present application), a simple integration such as

$$\Delta M_i = M_{i-1}\int_{t_{i-1}}^{t_i} \Omega\, dt = M_{i-1}\,\Delta\Theta \tag{1-14}$$

is sufficiently accurate for all practical purposes. The matrix $\Delta\Theta$ is defined by

$$\Delta\Theta = \begin{bmatrix} \Delta\theta_{11} & \Delta\theta_{12} & \Delta\theta_{13} \\ \Delta\theta_{21} & \Delta\theta_{22} & \Delta\theta_{23} \\ \Delta\theta_{31} & \Delta\theta_{32} & \Delta\theta_{33} \end{bmatrix} \tag{1-15}$$

where

$$\left.\begin{aligned}
&\Delta\theta_{11} = \Delta\theta_{22} = \Delta\theta_{33} = 0 \\
&\Delta\theta_{12} = -\Delta\theta_{21} = -\int_{t_{i-1}}^{t_i} \omega_{dw'}\, dt = -d_{16}\,\Delta t - d_{17}\,\Delta v'_{Tv} + d_{18}\,\Delta v'_{Tw} \\
&\Delta\theta_{13} = -\Delta\theta_{31} = \int_{t_{i-1}}^{t_i} \omega_{dv'}\, dt = -d_{13}\,\Delta t - d_{14}\,\Delta v'_{Tu} - d_{15}\,\Delta v'_{Tv} \\
&\Delta\theta_{23} = -\Delta\theta_{32} = -\int_{t_{i-1}}^{t_i} \omega_{du'}\, dt = -d_{10}\,\Delta t - d_{11}\,\Delta v'_{Tv} + d_{12}\,\Delta v'_{Tu}
\end{aligned}\right\} \tag{1-16}$$

and d_{10}, d_{11}, ..., d_{18} are the familiar gyro drift parameters.

The integration of the $\dot{M}$ matrix thus requires the calculation of $\Delta\theta_{12}$, $\Delta\theta_{13}$, and $\Delta\theta_{23}$ and then the matrix multiplication $M_{i-1}\,\Delta\Theta$. Using the notation

$$\left.\begin{aligned}
&M(t_{i-1}) = M_{i-1} = \left[m_{kj}\right] \\
&\text{and} \\
&\Delta M_i = \left[\Delta m_{kj}\right]
\end{aligned}\right\} \tag{1-17}$$

the following equations are obtained:

$$\left.\begin{aligned}
\Delta m_{11} &= -m_{12}\,\Delta\theta_{12} - m_{13}\,\Delta\theta_{13}\\
\Delta m_{12} &= m_{11}\,\Delta\theta_{12} - m_{13}\,\Delta\theta_{23}\\
\Delta m_{13} &= m_{11}\,\Delta\theta_{13} + m_{12}\,\Delta\theta_{23}\\
\\
\Delta m_{21} &= -m_{22}\,\Delta\theta_{12} - m_{23}\,\Delta\theta_{13}\\
\Delta m_{22} &= m_{21}\,\Delta\theta_{12} - m_{23}\,\Delta\theta_{23}\\
\Delta m_{23} &= m_{21}\,\Delta\theta_{13} + m_{22}\,\Delta\theta_{23}\\
\\
\Delta m_{31} &= -m_{32}\,\Delta\theta_{12} - m_{33}\,\Delta\theta_{13}\\
\Delta m_{32} &= m_{31}\,\Delta\theta_{12} - m_{33}\,\Delta\theta_{23}\\
\Delta m_{33} &= m_{31}\,\Delta\theta_{13} + m_{32}\,\Delta\theta_{23}
\end{aligned}\right\}\quad (1\text{-}18)$$

The calculation of ΔM by means of Equation 1-18 requires 18 distinct multiplications. Henceforth, this will be referred to as the "exact" transformation matrix. As will be shown later, this exact transformation is rather "expensive" from the standpoint of airborne-computer storage and compute-cycle requirements.

A significant saving in the required storage and cycle time can be achieved at the expense of a slight decrease in accuracy as follows. First, since the total platform drift angle will be relatively small, say less than five degrees, the diagonal elements of M do not differ appreciably from unity. Second, again as a consequence of the small angle consideration, the off-diagonal elements of M are approximately antisymmetric. From these considerations, the following "semiexact" calculation of ΔM can be achieved using only nine distinct multiplications.

$$\left.\begin{aligned}
\Delta m_{11} &= -m_{12}\,\Delta\theta_{12} - m_{13}\,\Delta\theta_{13}\\
\Delta m_{12} &= \Delta\theta_{12} - m_{13}\,\Delta\theta_{23}\\
\Delta m_{13} &= \Delta\theta_{13} + m_{12}\,\Delta\theta_{23}\\
\\
\Delta m_{21} &= -\Delta\theta_{12} - m_{23}\,\Delta\theta_{13}\\
\Delta m_{22} &= -m_{12}\,\Delta\theta_{12} - m_{23}\,\Delta\theta_{23}\\
\Delta m_{23} &= \Delta\theta_{23} - m_{12}\,\Delta\theta_{13}\\
\\
\Delta m_{31} &= -\Delta\theta_{13} + m_{23}\,\Delta\theta_{12}\\
\Delta m_{32} &= -\Delta\theta_{23} - m_{13}\,\Delta\theta_{12}\\
\Delta m_{33} &= -m_{13}\,\Delta\theta_{13} - m_{23}\,\Delta\theta_{23}
\end{aligned}\right\}\quad (1\text{-}19)$$

A further significant saving in the storage- and cycle-time requirements (at the expense of accuracy) can be achieved by retaining the second order contributions on the diagonal elements only. This method requires only three distinct multiplications. The calculation of ΔM now becomes,

$$\left.\begin{aligned}
\Delta m_{11} &= -m_{12}\,\Delta\theta_{12} - m_{13}\,\Delta\theta_{13}\\
\Delta m_{12} &= \Delta\theta_{12}\\
\Delta m_{13} &= \Delta\theta_{13}\\[1ex]
\Delta m_{21} &= -\Delta\theta_{12}\\
\Delta m_{22} &= -m_{12}\,\Delta\theta_{12} - m_{23}\,\Delta\theta_{23}\\
\Delta m_{23} &= \Delta\theta_{23}\\[1ex]
\Delta m_{31} &= -\Delta\theta_{13}\\
\Delta m_{32} &= -\Delta\theta_{23}\\
\Delta m_{33} &= -m_{13}\,\Delta\theta_{13} - m_{23}\,\Delta\theta_{23}
\end{aligned}\right\} \quad (1\text{-}20)$$

The ultimate simplified form of ΔM is obviously obtained by retaining only the off-diagonal terms. This is equivalent to assuming the total platform drift angle is of infinitesimal order. The result is

$$\left.\begin{aligned}
\Delta m_{11} &= \Delta m_{22} = \Delta m_{33} = 0\\
\Delta m_{12} &= -\Delta m_{21} = \Delta\theta_{12}\\
\Delta m_{13} &= -\Delta m_{31} = \Delta\theta_{13}\\
\Delta m_{23} &= -\Delta m_{32} = \Delta\theta_{23}
\end{aligned}\right\} \quad (1\text{-}21)$$

Thus, the calculation of ΔM by means of Equation 1-21 requires no multiplications. However, as will be shown, its accuracy is severely limited by the very small angle assumption.

The analytical drift compensations as discussed above, i.e., a) exact, b) semiexact, c) linear approximation with second order correction to diagonal elements only, and d) linear approximation, were programmed in closed-loop engineering guidance simulations on an IBM 7094 digital computer. Figure 1 presents a flow chart showing the "basics" as modified to accomodate Method 1.

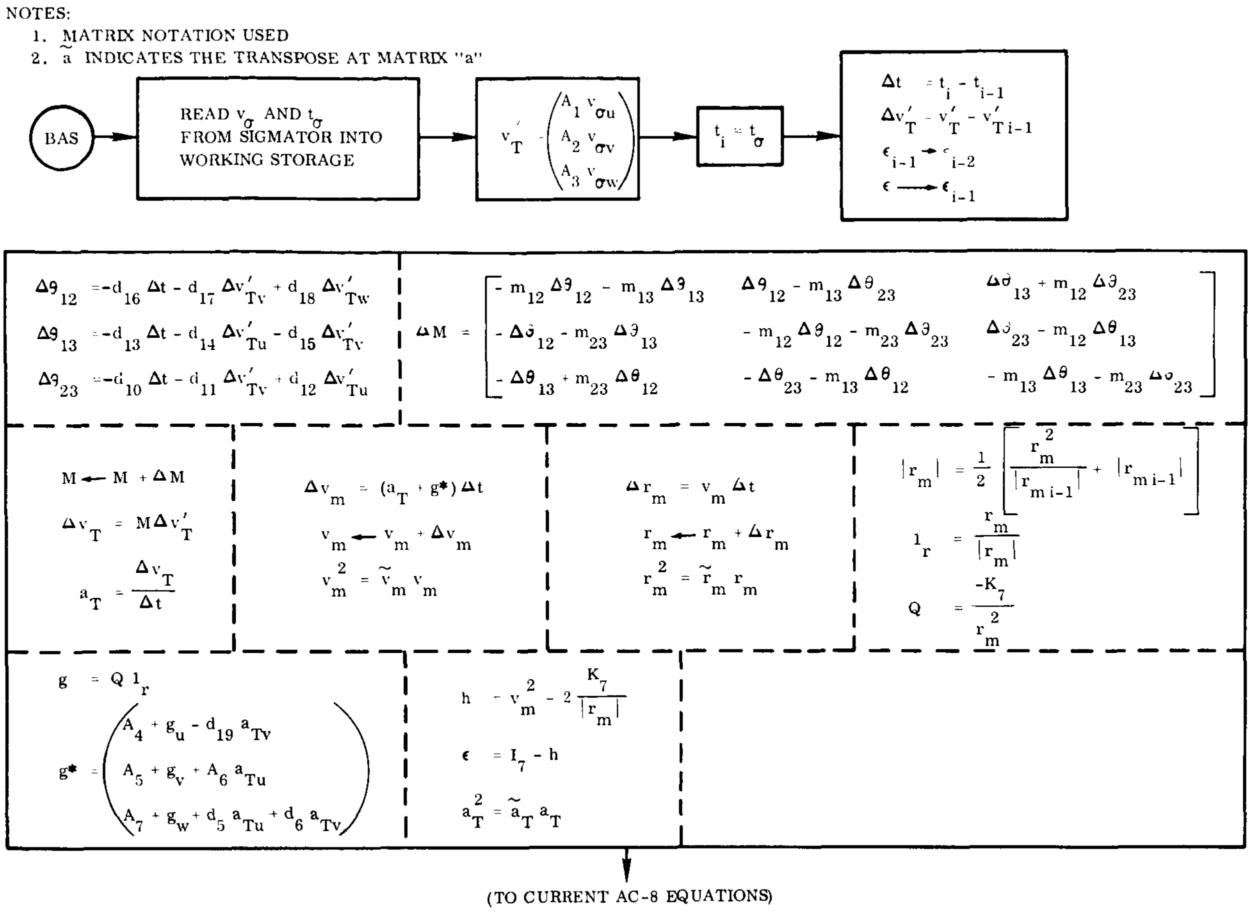

Figure 1. The Modified Basics for the Matrix Transformation Method

1.2 CONCLUSIONS. The accuracy of Method 1 was appraised by comparing the results obtained from nominal trajectories (d_{10}, d_{11}, . . . , and $d_{18} = 0$) with the results obtained from trajectories incorporating maximum-specification gyro drift rates, i.e., ± 3 deg/hr and ± 3 deg/hr/g. The following conclusions were reached:

a. Both the exact and semiexact matrix transformations result in negligible guidance errors.

b. The linear approximation incorporating second order corrections (quasi-linear approximation) to the diagonal elements only is adequate for direct-ascent missions. The resulting midcourse correction requirement (MCR) is approximately 1 m/sec.

c. The linear approximation is only adequate for gyro drift rates on the order of 1 deg/hr and 1 deg/hr/g or less. The residual error (MCR) increases approximately as the square of the gyro drift rates. Thus, if the MCR resulting from gyro drift rates of 1 deg/hr is 1 m/sec, the MCR resulting from 3 deg/hr drift rates would be about 9 m/sec.

SECTION 2

DRIFTED ROTATING-PLATFORM COORDINATE SYSTEM

2.1 DISCUSSION. The technique employed here is simply to perform the guidance computations in rotating (noninertial) platform coordinates. This reference frame is defined by the instantaneous orientation of the accelerometer input axes (corrected for nonorthogonality). The required kinematic relationships are

$$\left.\begin{aligned} \frac{d\bar{v}_m}{dt} &= \dot{\bar{v}}_m + \bar{\omega}_d \times \bar{v}_m \\ \frac{d\bar{r}_m}{dt} &= \dot{\bar{r}}_m + \bar{\omega}_d \times \bar{v}_m = \bar{v}_m \\ \frac{d\bar{1}_a}{dt} &= \dot{\bar{1}}_a + \bar{\omega}_d \times \bar{1}_a \end{aligned}\right\} \qquad (2\text{-}1)$$

where the operator $\frac{d}{dt}$ denotes differentiation with respect to fixed or inertial axes and the dot ($\cdot$) operator denotes differentiation with respect to the rotating platform axes. The angular velocity of the rotating-platform axes, $\bar{\omega}_d$, is given by

$$\bar{\omega}_d = \omega_{du'}\, \bar{1}_{u'} + \omega_{dv'}\, \bar{1}_{v'} + \omega_{dw'}\, \bar{1}_{w'} \qquad (2\text{-}2)$$

where

$$\left.\begin{aligned} \omega_{du'} &= d_{10} + d_{11}\, a_{Tv'} - d_{12}\, a_{Tu'} \\ \omega_{dv'} &= -d_{13} - d_{14}\, a_{Tu'} - d_{15}\, a_{Tv'} \\ \omega_{dw'} &= d_{16} + d_{17}\, a_{Tv'} - d_{18}\, a_{Tw'} \end{aligned}\right\} \qquad (2\text{-}3)$$

The total vehicle acceleration, $\frac{d\bar{v}_m}{dt}$, is given by

$$\frac{d\bar{v}_m}{dt} = \bar{a}_T + \bar{g}^* \qquad (2\text{-}4)$$

where $\bar{a}_T$ is the thrust acceleration and $\bar{g}^*$ is the gravitational acceleration, including the effects of accelerometer bias and input axis misalignment. The unit target vector as seen by the inertial observer is fixed (constant) in direction. Therefore, $\frac{d\bar{1}_a}{dt} = 0$. Equation 2-1 can now be expressed as

$$
\left.\begin{aligned}
\dot{\bar{v}}_m &= \bar{a}_T + \bar{g}^* - \bar{\omega}_d \times \bar{v}_m \\
\dot{\bar{r}}_m &= \bar{v}_m - \bar{\omega}_d \times \bar{r}_m \\
\dot{\bar{1}}_a &= - \bar{\omega}_d \times \bar{1}_a
\end{aligned}\right\} \qquad (2\text{-}5)
$$

It will be noted here that the three vector cross products in Equation 2-5 require eighteen distinct multiplications. It will be recalled that this is the same number of discrete multiplications required in the exact calculation of the ΔM matrix. (See Equation 1-18.) However, Method 1 requires an additional matrix vector multiplication $(M(t_i)\Delta v'_{Ti})$ which necessitates nine further discrete multiplications! Note also that the vector components of $\bar{a}_T$ are inherently in the correct coordinate system and can therefore be utilized directly; e.g., the integral of $\bar{a}_T$ (performed automatically by the integrating (pulse-rebalanced) accelerometers) is exactly what is required for integrating the first of Equations 2-5 in the rotating-platform coordinate system.

This technique of analytically compensating for gyro drift was programmed in a closed-loop engineering guidance simulation mechanized on the IBM 7094 digital computer. Figure 2 is a flow chart showing the modified basics required for this scheme.

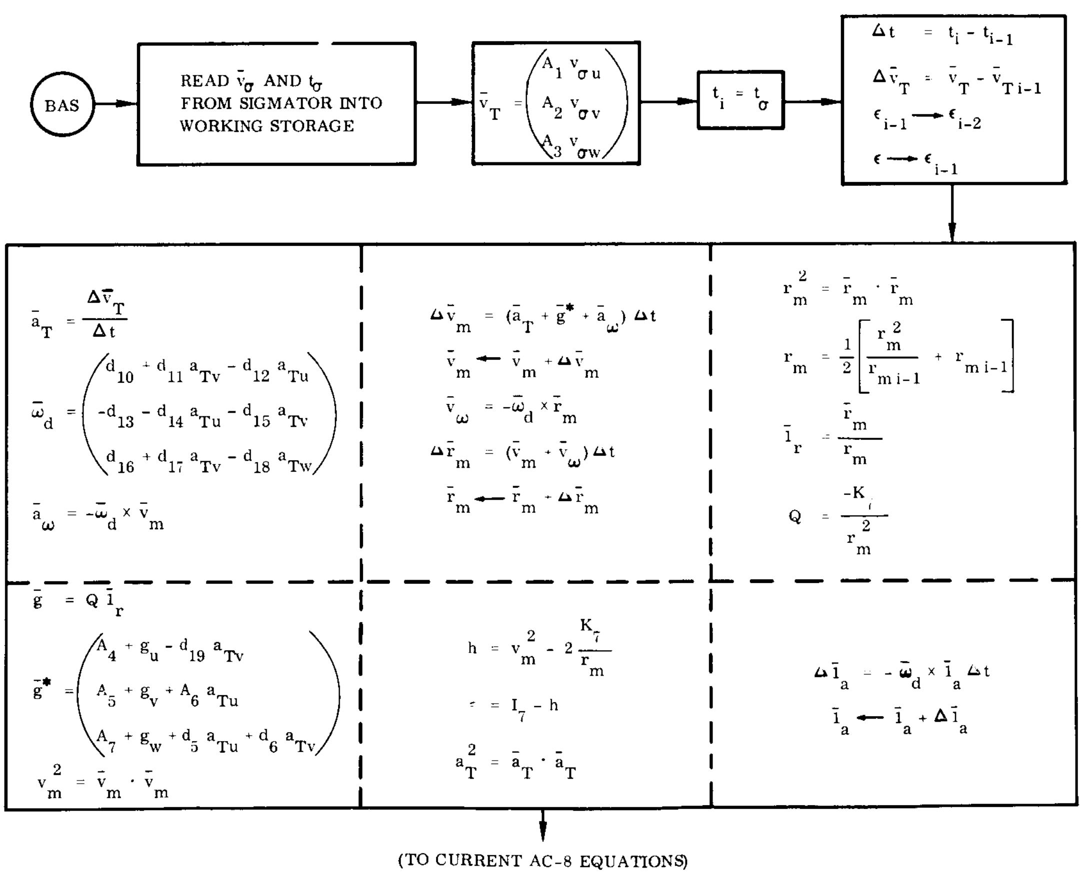

Figure 2. The Modified Basics for the Drifted Rotating-Platform Coordinate System Method

2.2 CONCLUSIONS. This technique is very accurate and relatively economical for airborne-computer requirements, Guidance system inaccuracy attributable to this scheme is essentially zero. Furthermore, from a practical point of view, there appears to be no limitations on its applicability. For example, there are no small-angle assumptions involved. Intuitively, it seems that the optimum reference frame for navigational computation would be the reference frame in which the thrust acceleration is sensed.

SECTION 3

ROTATING t, n, r COORDINATE SYSTEM

3.1 DISCUSSION. The technique employed here is fundamentally similar to that discussed in Section 2. That is, the navigational calculations are again performed in a rotating (noninertial) coordinate system. This coordinate system, herein referred to as the t, n, r coordinate system, is defined by the instantaneous position and velocity of the vehicle. The orientation of the t, n, r axes is defined by the unit vectors $\bar{1}_r$, $\bar{1}_n$, and $\bar{1}_t$ as follows:

$$\left.\begin{aligned} \bar{1}_r &= \frac{\bar{r}}{r} \\ \bar{1}_n &= \frac{\bar{1}_r \times \bar{v}}{|\bar{1}_r \times \bar{v}|} \\ \bar{1}_t &= \bar{1}_n \times \bar{1}_r \end{aligned}\right\} \qquad (3\text{-}1)$$

Note that the position vector has only one nonzero component when referred to the t, n, r axes. Thus,

$$\bar{r} = r\,\bar{1}_r \qquad (3\text{-}2)$$

Further, note that the velocity vector has only two nonzero components when referred to the t, n, r axes. Thus,

$$\bar{v} = v_r\,\bar{1}_r + v_t\,\bar{1}_t \qquad (3\text{-}3)$$

The required kinematic relationships are

$$\left.\begin{aligned} \frac{d\bar{v}}{dt} &= \dot{\bar{v}} + \bar{\omega} \times \bar{v} \\ \frac{d\bar{r}}{dt} &= \dot{\bar{r}} + \bar{\omega} \times \bar{r} \\ \frac{d\bar{1}_a}{dt} &= \dot{\bar{1}}_a + \bar{\omega} \times \bar{1}_a \end{aligned}\right\} \qquad (3\text{-}4)$$

where the operator $\frac{d}{dt}$ denotes differentiation with respect to fixed or inertial axes and the dot $(\dot{\ })$ operator denotes differentiation with respect to the rotating t, n, r axes. Here, $\bar{\omega}$ is the angular velocity of the t, n, r reference frame with respect to an inertial frame. Expressed in t, n, r coordinates, $\bar{\omega}$ is given by

$$\bar{\omega} = \omega_r \bar{1}_r + \omega_t \bar{1}_t + \omega_n \bar{1}_n \qquad (3\text{-}5)$$

The components of the angular velocity vector $\bar{\omega}$ can be determined as follows. Differentiate the first of Equations 3-1,

$$\frac{d\bar{1}_r}{dt} = \frac{1}{r}\frac{d\bar{r}}{dt} - \frac{\bar{r}}{r^2}\frac{dr}{dt}$$

$$= \frac{v_t}{r}\bar{1}_t \qquad (3\text{-}6)$$

Now, the derivative $\frac{d\bar{1}_r}{dt}$ is also given by

$$\frac{d\bar{1}_r}{dt} = \dot{\bar{1}}_r + \bar{\omega} \times \bar{1}_r$$

$$= \bar{\omega} \times \bar{1}_r$$

$$= \omega_n \bar{1}_t - \omega_t \bar{1}_n \qquad (3\text{-}7)$$

Equating coefficients in Equations 3-6 and 3-7, one obtains

$$\left.\begin{aligned} \omega_n &= \frac{v_t}{r} \\ \omega_t &= 0 \end{aligned}\right\} \qquad (3\text{-}8)$$

Now, differentiate the second of Equations 3-1,

$$\frac{d\bar{1}_n}{dt} = \frac{\frac{d\bar{1}_r}{dt} \times \bar{v} + \bar{1}_r \times \frac{d\bar{v}}{dt}}{|\bar{1}_r \times \bar{v}|} - \frac{\bar{1}_r \times \bar{v}}{|\bar{1}_r \times \bar{v}|^2}\frac{d}{dt}|\bar{1}_r \times \bar{v}|$$

$$= \frac{1}{|\bar{1}_r \times \bar{v}|}\left[\frac{d\bar{1}_r}{dt} \times \bar{v} + \bar{1}_r \times \frac{d\bar{v}}{dt} - \bar{1}_n \frac{d}{dt}|\bar{1}_r \times \bar{v}|\right] \qquad (3\text{-}9)$$

The derivative of $|\bar{1}_r \times \bar{v}|$ can be calculated as

$$\frac{d}{dt}\left|\bar{1}_r \times \bar{v}\right| = \frac{d}{dt}\left[(\bar{1}_r \times \bar{v}) \cdot (\bar{1}_r \times \bar{v})\right]^{1/2}$$

$$= \frac{1}{2}\,\frac{2(\bar{1}_r \times \bar{v}) \cdot \frac{d}{dt}(\bar{1}_r \times \bar{v})}{\left|\bar{1}_r \times \bar{v}\right|}$$

$$= \bar{1}_n \cdot \frac{d}{dt}(\bar{1}_r \times \bar{v})$$

$$= \bar{1}_n \cdot \left(\frac{d\bar{1}_r}{dt} \times \bar{v} + \bar{1}_r \times \frac{d\bar{v}}{dt}\right) \tag{3-10}$$

Substituting this result into the right-hand side of Equation 3-9, one obtains

$$\frac{d\bar{1}_n}{dt} = \frac{1}{\left|\bar{1}_r \times \bar{v}\right|}\left\{(I - \bar{1}_n\bar{1}_n) \cdot \left(\frac{d\bar{1}_r}{dt} \times \bar{v} + \bar{1}_r \times \frac{d\bar{v}}{dt}\right)\right\} \tag{3-11}$$

where $I = \bar{1}_t\bar{1}_t + \bar{1}_n\bar{1}_n + \bar{1}_r\bar{1}_r$ is the unit dyad.

Now,

$$\left.\begin{aligned}
\frac{d\bar{1}_r}{dt} \times \bar{v} &= \frac{v_t}{r}\bar{1}_t \times (v_t\bar{1}_t + v_r\bar{1}_r) = \frac{-v_t v_r}{r}\bar{1}_n \\
\bar{1}_r \times \frac{d\bar{v}}{dt} &= \bar{1}_r \times (\bar{a}_T + \bar{g}) = -a_{Tn}\bar{1}_t + a_{Tt}\bar{1}_n \\
\left|\bar{1}_r \times \bar{v}\right| &= v_t
\end{aligned}\right\} \tag{3-12}$$

which, upon substituting into the right hand member of Equation 3-11, yields

$$\frac{d\bar{1}_n}{dt} = -\frac{a_{Tn}}{v_t}\bar{1}_t \tag{3-13}$$

Also, since

$$\frac{d\bar{1}_n}{dt} = \bar{\omega} \times \bar{1}_n$$

$$= -\omega_r\bar{1}_t \tag{3-14}$$

then

$$\omega_r = \frac{a_{Tn}}{v_t} \tag{3-15}$$

Thus, the components of $\bar{\omega}$ may be expressed as

$$\left.\begin{aligned} \omega_t &= 0 \\ \omega_n &= \frac{v_t}{r} \\ \omega_r &= \frac{a_{Tn}}{v_t} \end{aligned}\right\} \tag{3-16}$$

Using Equations 3-16, Equations 3-4 may be written in the following scalar form:

$$\left.\begin{aligned} \dot{v}_t &= a_{Tt} - \omega_n v_r \\ \dot{v}_r &= a_{Tr} - g + \omega_n v_t \\ \dot{r} &= v_r \\ \dot{1}_{at} &= \omega_r 1_{an} - \omega_n 1_{ar} \\ \dot{1}_{an} &= -\omega_r \bar{1}_{at} \\ \dot{1}_{ar} &= \omega_n 1_{at} \end{aligned}\right\} \tag{3-17}$$

Before integrating Equations 3-17, it is necessary to determine the thrust acceleration vector $\bar{a}_T$ referred to the t, n, r coordinate system. Therefore, we require a transformation matrix which will transform the thrust acceleration vector from platform coordinates (where it is actually measured) to t, n, r coordinates. This $\widetilde{M}$ matrix is obtained in the same manner as was explained in detail above for Method 1. Thus Equation 1-18 is also applicable here if we define the $\Delta\theta$'s as follows:

$$\left.\begin{aligned} \Delta\theta_{12} &= -(\omega_r - \omega_{dr})\,\Delta t \\ \Delta\theta_{13} &= (\omega_n - \omega_{dn})\,\Delta t \\ \Delta\theta_{23} &= \omega_{dt}\,\Delta t \end{aligned}\right\} \tag{3-18}$$

ω_{dt}, ω_{dn}, and ω_{dr} are the components of the platform drift rate $\bar{\omega}_d$ referred to the t, n, r coordinate axes. Using matrix notation,

$$\omega_d = \begin{bmatrix} \omega_{dt} \\ \omega_{dn} \\ \omega_{dr} \end{bmatrix} = \tilde{M} \omega_d^* \qquad (3-19)$$

where

$$\omega_d^* = \begin{bmatrix} d_{10} + d_{11} a_{Tv}^* - d_{12} a_{Tu}^* \\ -d_{13} - d_{14} a_{Tu}^* - d_{15} a_{Tv}^* \\ d_{16} + d_{17} a_{Tv}^* - d_{18} a_{Tw}^* \end{bmatrix} \qquad (3-20)$$

The acceleration components a_{Tu}^*, a_{Tv}^*, and a_{Tw}^* are the thrust acceleration components (platform axes), corrected for scale factor, bias, and misalignments.

Finally, it must be noted that the reference attitude vector $\bar{f}^*$, being calculated in t, n, r coordinates, must ultimately be transformed back to platform coordinates in order to steer the vehicle.

This method was not programmed since it was fairly obvious that the airborne-computer storage requirements would be excessive. However, the storage requirements and compute-cycle degradation were accurately estimated for comparison purposes. Figure 3 presents a flow chart reflecting the modified basics used for the rotating t, n, r coordinate system navigational computations.

3.2 CONCLUSIONS. This method, although probably workable, results in almost precisely the same airborne-computer storage and compute-cycle penalties as does the exact matrix transformation (Method 1). The integration of the equations of motion and the attitude reference vector calculations are much simpler than the corresponding calculations required in the other methods. However, since the accelerometer outputs must be transformed from the drifting platform reference axes to the rotating t, n, r coordinate system (and back again for steering!), there is a significant net penalty involved.

BASIC EQUATIONS

BAS → READ v_σ AND t_σ FROM SIGMATOR INTO WORKING STORAGE → $v'_T = \begin{pmatrix} A_1 v_{\sigma u} \\ A_2 v_{\sigma v} \\ A_3 v_{\sigma w} \end{pmatrix}$ → $t_i = t_\sigma$ →

$$\Delta t = t_i - t_{i-1}$$
$$\Delta v'_T = v'_T - v'_{T\,i-1}$$
$$\epsilon_{i-1} \longrightarrow \epsilon_{i-2}$$
$$\epsilon \longrightarrow \epsilon_{i-1}$$

$$a'_T = \frac{\Delta v'_T}{\Delta t}$$

$$a^*_T = \begin{pmatrix} a'_{Tu} + A_4 - d_{19}\, a'_{Tv} \\ a'_{Tv} + A_5 + A_6\, a'_{Tu} \\ a'_{Tw} + A_7 + d_5\, a'_{Tu} + d_6\, a'_{Tv} \end{pmatrix}$$

$$\Delta\theta_{12} = -(\omega_r - \omega_{dr})\,\Delta t \quad ; \quad \Delta\theta_{23} = \omega_{dt}\,\Delta t$$

$$\Delta\theta_{13} = (\omega_n - \omega_{dn})\,\Delta t$$

$$\Delta M = \begin{bmatrix} -m_{12}\Delta\theta_{12} - m_{13}\Delta\theta_{13} & m_{11}\Delta\theta_{12} - m_{13}\Delta\theta_{23} & m_{11}\Delta\theta_{13} + m_{12}\Delta\theta_{23} \\ -m_{22}\Delta\theta_{12} - m_{23}\Delta\theta_{13} & m_{21}\Delta\theta_{12} - m_{23}\Delta\theta_{23} & m_{21}\Delta\theta_{13} + m_{22}\Delta\theta_{23} \\ -m_{32}\Delta\theta_{12} - m_{33}\Delta\theta_{13} & m_{31}\Delta\theta_{12} - m_{33}\Delta\theta_{23} & m_{31}\Delta\theta_{13} + m_{32}\Delta\theta_{23} \end{bmatrix}$$

$$M \longleftarrow M + \Delta M$$

$$a_T = \begin{pmatrix} a_{Tt} \\ a_{Tn} \\ a_{Tr} \end{pmatrix} = \tilde{M}\, a^*_T$$

$$\Delta r = v_r\,\Delta t$$

$$r \longleftarrow r - \Delta r$$

$$g = \frac{K_7}{r^2}$$

$$\Delta v_r = \left(a_{Tr} - g + \frac{v_t^2}{r}\right)\Delta t$$

$$v_r \longleftarrow v_r + \Delta v_r$$

$$\Delta v_t = \left(a_{Tt} - \frac{v_r v_t}{r}\right)\Delta t$$

$$v_t \longleftarrow v_t + \Delta v_t$$

$$\omega_n = \frac{v_t}{r}$$

$$\omega_r = \frac{a_{Tn}}{v_t}$$

$$v^2 = v_r^2 + v_t^2$$

$$h = v^2 - 2\frac{K_7}{r}$$

$$\epsilon = I_7 - h$$

$$a_T^2 = \tilde{a}_T\, a_T$$

$$\omega^*_d = \begin{pmatrix} d_{10} + d_{11} a^*_{Tv} - d_{12} a^*_{Tu} \\ -d_{13} - d_{14} a^*_{Tu} - d_{15} a^*_{Tv} \\ d_{16} + d_{17} a^*_{Tv} - d_{18} a^*_{Tw} \end{pmatrix}$$

$$\omega_d = \begin{pmatrix} \omega_{dt} \\ \omega_{dn} \\ \omega_{dr} \end{pmatrix} = \tilde{M}\, \omega^*_d$$

$$\Delta 1_a = \begin{pmatrix} (\omega_r 1_{an} - \omega_n 1_{ar})\,\Delta t \\ -\omega_r 1_{at}\,\Delta t \\ \omega_n 1_{at}\,\Delta t \end{pmatrix}$$

$$1_a \longleftarrow 1_a + \Delta 1_a$$

$$\dot{\epsilon}_y = v_t 1_{an}$$

$$m = 1_{at}$$

$$|a_T| = \frac{1}{2}\left[\frac{a_T^2}{|a_{T\,i-1}|} + |a_{T\,i-1}|\right]$$

$$1_{aT} = \frac{a_T}{|a_T|}$$

Figure 3. The Modified Basics for the Rotating t, n, r Coordinate System Method

STEERING

$$V_{rn} = \begin{cases} \dfrac{K_8 \epsilon \dot{\epsilon}_y}{m} \; ; & P_8 = 1 \\ K_9 m \epsilon \dot{\epsilon}_y \; ; & P_8 = 0 \end{cases}$$

$$r^* = r - C_2$$

$$m^* = m - C_1$$

$$V_{rr} = C_3 + C_4 r^* + C_5 m^* + C_6 m^{*2}$$

$$= 0 \text{ for two burn}$$

$$V_{rt}^2 = h_d + 2 \frac{K_7}{r} - V_{rr}^2$$

$$V_{rt} = \frac{1}{2} \left[\frac{V_{rt}^2}{V_{rt\,i-1}} + V_{rt\,i-1} \right]$$

$$f = \begin{pmatrix} V_{rt} - v_t \\ V_{rn} \\ V_{rr} + G_r - v_r \end{pmatrix}$$

$$f^2 = \tilde{f}\, f$$

$$|f| = \frac{1}{2} \left[\frac{f^2}{|f_{i-1}|} + |f_{i-1}| \right]$$

$$1_f = \frac{f}{|f|}$$

$$\Delta f \leftarrow \Delta f + K_{20} (1_f - 1_{aT}) \Delta t$$

$$f^{*\prime} = K_{21} (1_f + \Delta f)$$

$$f^* = M f^{*\prime}$$

(TO CURRENT AC-8 EQUATIONS)

Figure 3. The Modified Basics for the Rotating t, n, r Coordinate System Method (Continued)

SECTION 4

GUIDANCE PARAMETER BIASING

The method employed here is essentially identical to the current technique of biasing the trajectory dependent guidance parameters to account for the effects of the oblateness of the earth, etc. In effect, targeting is done using a drifting (rotating) platform. Thus, the polynomials for the calculation of 1_{au}, 1_{av}, 1_{aw}, h_d, C_1, . . . , and C_5 are also functions of d_{10}, d_{11}, . . . , and d_{18}.

The feasibility of this method was analyzed and verified by employing an engineering closed-loop guidance simulation mechanized on the IBM 7094 digital computer. This technique, although perfectly accurate, is quite "expensive" from the standpoint of additional targeting and the subsequent curve fitting required.

In addition, the airborne-computer storage penalty is rather severe. In fact, it appears that the only distinct advantage to be offered by this method is that the inflight compute cycle remains unchanged.

SECTION 5

CONCLUSIONS AND RECOMMENDATIONS

Each of the various proposed techniques for analytically compensating for the effects of inflight gyro drift rates was verified to be feasible. Also, there does not appear to be any significant trade-off as far as accuracy is concerned since the residual error (MCR) resulting from the use of each method is essentially zero (the quasi-linear matrix transformation method is an exception for worst-case gyro drift rates). Thus, the ultimate basis for comparison of the different methods should be based on their respective airborne-computer programming requirements. The following table presents a summary of the inflight-storage and compute-cycle penalties corresponding to each method using the current AC-8 guidance equations as a common reference.

METHOD	INCREASE IN DRUM REVOLUTIONS	INCREASE IN CELLS	REMARKS
1 (Exact)	10.9	64	
1(Semiexact)	6.2	46	
1(Quasi-linear)	2.6	8	Includes second order corrections to diagonal elements only. Limited to very small angles.
2	6.2	16	
3	11.5	63	
4	0	>162	Does not include the additional cells required for numerically evaluating polynomials!

It should be pointed out that the estimates presented above assume perfect programming optimization. The actual values will be somewhat higher. The comparison, however, is still valid.

From the results presented in the table, it is apparent that the technique employing the quasi-linear matrix transformation requires minimum airborne computer storage and results in the shortest inflight compute-cycle length. However, the requirements utilizing Method 2, the drifted rotating-platform coordinate system, are not significantly different. In view of this fact and also since Method 2 consitutes an exact method, it is recommended that Method 2 be chosen as the best technique for analytical inflight gyro drift rate compensation.